Mathematical Finance, Information Flow, and Economic Growth in pre-industrial Europe

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Abstract

This paper studies the role of development in financial techniques in the economic growth in Europe from the thirteenth to the seventeenth century. During the era mathematicians developed the main advances in finance, and Fibonacci's book *Liber Abaci* is undoubtedly the most important development. This paper uses the publications of mathematics books as a measure of exposure to new financial techniques, and exploits city-level data. Results suggest that the of exposure to new financial technologies had a causal effect in economic growth before the sixteenth century. The presence of reverse causality and measurement error after the invention of the printing press make the effect impossible to identify after the sixteenth century.

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1 Introduction

One of the oldest themes in economics is the one about technological change. The difference between technological change (inspiration) and resources accumulation (transpiration) as the source of economic development is fundamental in the study of economic growth. More productive economies would need less resources to produce the same output than less productive ones. Or equivalently, if two economies have the same resources the one that is more productive would have a higher quality of living. This question goes back to the Ancient Greek: in his *Politics*, Aristotle defends private property over Plato's idea of common property of the ruling classes only because he believed that private administration was more productive. Moreover, productivity was the main reason why Adam Smith argued in favor of the division of labor

> The greatest improvement in the productive powers of labour, and the greater part of the skill, dexterity, and judgment with which it is anywhere directed, or applied, seem to have been the effects of the division of labour[1].

Economists have spent a considerable amount of effort[2] thinking about productivity and how technology affects the way in which factors are used to to produce physical goods or services. Among these changes in productivity, ideas have been identified as one of the key components (Romer (1990), Lucas (2009), Jones and Romer (2010)). This paper addresses a different kind of technology, and therefore a different kind of idea. Instead of thinking about ideas that increase the level of product generated with a specific level of factors, I focus on ideas that increase the return of capital without changing the productivity. I will refer generically to these ideas as changes in financial technology.

Nowadays the idea of important differences in financial technology is not plausible. Asset pricing techniques are standard across investors, and information flows faster than ever[3]. However this was not always the case: early developments in financial mathematics and asset pricing techniques took place even before the invention of the

[1] Smith (1976)

[2] See Syverson (2011) for a survey of the literature

[3] It is possible that new developments in information technologies and high-frequency trading are again generating differences in investment technology, as argued by Budish, Cramton, and Shim (2015).

printing press. In those years it was likely that new financial techniques adopted in one place were not replicated by others. This would generate differences in standard of living due productivity outside the typical production function. This paper address this differences in financial technologies and their effect of economic growth before the industrial revolution across European cities.

Probably the most revolutionary ideas in finance before the industrial revolution (if not in the history of the human kind) are the ones presented in the book *Liber Abaci* (The Books of Calculations) written by Leonardo de Pisa, commonly known as Fibonacci, and published in 1202. Although this book is mostly known by introducing the Hindu-Arabic numeric system to Europe and nowadays by the Fibonacci's sequence[4], it also includes novel techniques to compute operations with fractions, and financial elements as exchange rates, interest rates and present values.

The Book of Calculations is arguably the first book to develop a mathematical approach to financial calculations. In a world in which the most sophisticated computation machine is an abacus the problem of even converting a fraction to decimal numbers can become a non-trivial one, and Fibonacci developed techniques to solve this and what he claimed "nearly all problems in mathematics"[5]. This could have generated comparative differences for investment and trade in favor of the people that knew these techniques, and could be a reason of the unprecedented economic development of Italy from 1200 to 1500 (see for example Maddison (2010), Malanima (2011)). However, to the best of my knowledge there have been no attempts to quantitatively evaluate the effect of this ideas on economic development.

Fibonacci was a mathematician. And as many mathematicians of his age, he was interested in commercial problems. In a world not only without internet, but even without the printing press, the spread of Fibonacci's ideas through Europe must have been mainly through mathematicians interested in commercial applications and the books they wrote. This paper explores that specific idea to address the effect of different

[4]The sequence defined by its first number being zero, its secon number being one, and each number after that being the sum of the two previous numbers, $a_n = a_{n-1} + a_{n-2}$. This sequence has the property that in the limit the ratio between a number and the previous one is equal to the golden ratio $\frac{1+\sqrt{5}}{2}$.

[5]The 15^{th} chapter of the book is entitled "On the Method Elchatayam and How With It Nearly All Problems in Mathematics are Solved". For more deltails on this method see Maruszewski (2009).

investment technologies over economic growth. For this, I gather data on mathematical books published in different European cities and analyze how they influence those cities economic level (measured as population).

Results show that there is a direct relation between number of published books in mathematics and economic development for the 1200-1700 period. However, falsification exercises show that this relation can be interpreted as a causal effect of adoption of financial technologies only before the sixteenth century.

With the invention of the printing press, the dynamics under books were produced and traded changed completely during the sixteenth century. Since the printing press increased the number and availability of books, it generated a new market: the market for books. Book trade in Europe started during the sixteenth century (see for example Pollard (1916) or Pottinger (1958)). The increase in book production and trade make the number of published books in each city a less likely measure of exposure to financial technologies, both because books were less likely to stay in the city they were published and because the high initial costs of acquiring the printing press technology make the relation between income and book publications impossible to interpret.

The paper proceeds as follows. Section 2 describes the book *Liber Abaci* and the advances in financial technologies that it introduced. Section 3 describes the data, and Section 4 presents the empirical strategy. Section 5 presents the main results. Finally, Section 6 concludes.

2 Fibonacci's Book of Calculations

Consider the following problem

> A hundred pound of pepper is worth 13 pounds, and a hundredweight of cinnamon is worth 3 pounds; it is sought how many rolls of cinnamon are had for 342 pounds of pepper[6].

Although today this is considered an extremely easy exercise for an average businessman, this was not always the case. It is taken from the *Liber Abaci*, where Fibonacci uses it as an example to describe the technique he developed to solve this kind

[6] Sigler (2003), p. 181.

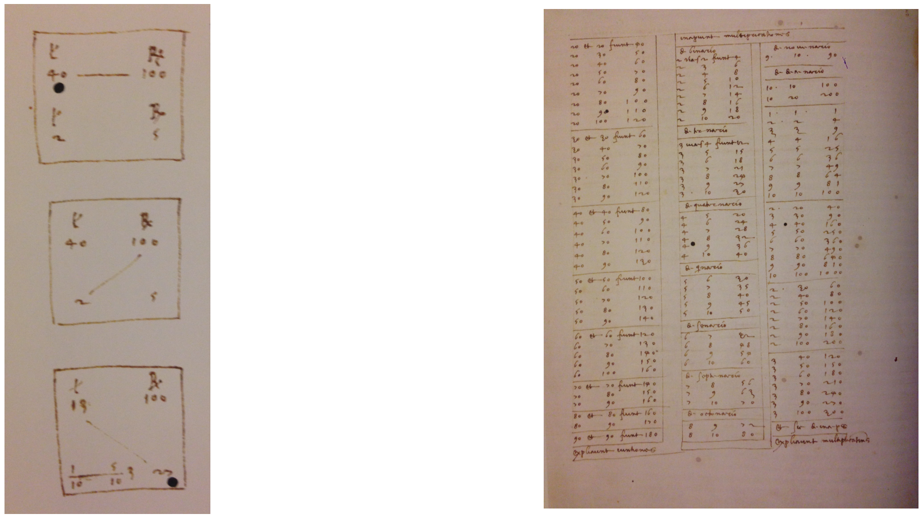
of problems.

Fibonacci (1775-1250) was the son of an administrative official in the Pisan colony of Bugia, in North Africa. According to his father's request he was trained in Arabic mathematical methods, as he describes in his book. He also mention several travels during his youth, where he acquired mathematical knowledge from "whoever was learned in it, from nearby Egypt, Syria, Greece, Sicily and Provence, and their various methods, to which locations of business I travelled considerably afterwards for much study"[7]. He gives considerable credit throughout his book to the work of Muhhammad ibn Mūsi Al Khwārizmī, whose ideas knew most likely through his famous treatise *The Algebra*.

The book of calculations has fifteen different chapters. It begins with an exposition of the fundamentals of arithmetic: multiplication, addition, subtraction, and division (in that order). After that, it revises the same operations in the case of numbers with fractions. Since in the thirteenth century Europe still used the Roman number system, this chapters were extremely important for commercial uses. Figure 2 presents in detail two important financial tools from the arithmetic section of the book: panel a) shows the application of the rule of three to compute operations with fractions, and panel b) shows the table of products in *Liber Abaci*. This table is arguably the earliest Arabic numeral multiplication table in Western mathematics, therefore a fundamental tool for rapid financial calculation.

After the description of arithmetic operations, Fibonacci spend several chapters on financial problems. Specifically, the chapters eight to eleven are entitled "On Finding the Value of Merchandise and Similar Things", "On the Bartering of Merchandise and Similar Things", "On Companies and Their Members", and "On the Alloying of Money". The contents regarding financial technology can be classified in five categories according to Goetzmann and Rouwenhorst (2005): foreign exchange and coinage, division of profits, traveling merchant problems, interest rate and baking problems, and present value analysis. Figure 2 presents notes on the solution of the problem presented at the beginning of this section (panel a) and for the computation of present values in the following problem (panel b)

[7] Sigler (2003), p. 15.



(a) Application of the Rule of Three

(b) Multiplication Table

Figure 1: Notes on Arithmetic Operations in *Liber Abaci*

> A certain man placed 100 pounds at a certain house for IIII denari per pound per month interest, and he took back in each year a payment of 30 pounds; one must compute in each year the 30 pounds reduction of capital and the profit on the said 100 pounds. It is sought how many years, months, days, and hours he will hold money in the house[8].

Both figures are taken from Sigler (2003)'s version of *Liber Abaci.* It can be seen in panel b) that the write to write fractions that Fibonacci used is significantly different than the one commonly used. For a more detailed description of the book and the techniques developed in it see Goetzmann (2004).

rolls of cinnamon		*pounds [value]*		*pounds of pepper*
1482		13		100
	*		*	
	*		*	
100		3		342

(a) Barter of Common Things

	10
	12
	$\frac{2}{5}14$
	$\frac{2\ 1}{5\ 5}17$
	$\frac{2\ 3\ 3}{5\ 5\ 5}20$
$\frac{2\ 2\ 2\ 1}{5\ 5\ 5\ 5}99$	$\frac{2\ 0\ 2\ 4}{5\ 5\ 5\ 5}24$

(b) Present Value

Figure 2: Notes on Financial Calculations in *Liber Abaci*

[8] Sigler (2003), p. 384.

Academics in finance, history, and mathematics coincide in the idea that Fibonacci's contribution is one of the most important, if not the most important, in the history of finance. Akyıldırım and Soner (2014) argue that Fibonacci is the first big contributor in the history of financial mathematics since Thales, and writes that although he is "one of the most famous names in mathematics regarding his contributions to number theory and other related areas [...] he may be considered even more influential in finance because of his contributions to the foundations of credit and banking in Europe through present value computations" (p. 2). Some others like Goetzmann (2004) are argue further. He presents Fibonacci's ideas as the main cause of the great divergence between Europe and Asia in the 19th century. Although this thesis seems extreme, the fact that the Great Divergence was generated by trade development (see Acemoglu, Johnson, and Robinson (2005)) suggests that Fibonacci's financial developments were relevant in this process.

3 Data Description

3.1 Economic Development

One of the challenges of empirical work when trying to answer big historical questions is the lack of data availability. Specifically, there are no details about economic development for the pre-Industrial Revolution era. Broadberry (2013) assembles several databases and build estimates of per capita GDP for four European countries and three Asian ones. However, the geographical extensions of countries and the differences currencies within each one makes a country level measure unreliable for the purposes of this research.

Given the unavailability of data on economic prosperity a widely used practice is to measure economic development with population. As an example, De Long and Shleifer (1993) argue that although some rich European cities were "centers of neither trade nor urban industry but instead the home of bureaucrats and the favored dwelling places of the landlord", these were exceptions and not the general rule. They state that the general pre-industrial European city was primarily a center of commerce, and "the growth of agricultural productivity and economic specialization had advanced far enough to support them". Given this, city size can be a good indicator of economic development.

Since Fibonacci's book was published in the year 1202, an ideal database would have population information from the year 1200 onwards. The only database with that level of detail that I am aware of is the one constructed by Bairoch, Batou, and Pierre (1988). This data estimates city population for the years 800 to 1850, but as the focus of the research is on pre-industrial growth, I only use data until 1700 for the analysis. The authors began with cities' estimated sizes provided by Tertius Chandler and Gerald Fox, and extended and updated the database for more than a decade. A comparison between Bairoch's database and alternative estimations in De Long and Shleifer (1993) support the reliability of the data.

3.2 Financial Technology

Although the Hindu-Arabic numerical system became the main system only in the 16th century (Merzbach and Boyer (2011)), algorithmic methods based on the Hindu-Arabic numerals and the use of the "bill of exchange" (which enabled an exporter of goods to receive payment in his own currency) were used at the higher levels of banking (Biggs (2009)) even in the fourteenth century. The use of these techniques near Fibonacci's hometown Pisa may help to explain the unprecedented economic development in Italy between 1200 and 1500.

Motivated by Chaney (2016), I measure the extension of Fibonacci's ideas among different cities in Europe by using the number of books about mathematics published in different cities for different centuries. I build this dataset using the registry from the Library of the University of California, Berkeley. It is reasonable to assume that UC Berkeley Library provides a good approximation to the number of relevant books in mathematics published in Europe in the pre-industrial era. First, this registry includes the books available in the twenty-three libraries across the campus, besides registry of all the books that are available only as electronic resources. According to Bland, Kyrillidou, et al. (2009), as of 2006 the library contained more than 10 million volumes, making it the fourth largest university library in the United States, surpassed only by the libraries of Harvard, Yale, and the University of Illinois at Urbana-Champaign.

Second, UC Berkeley's famous strength in the study of mathematics makes it likely that the most relevant works in the area of mathematics will be presented in the database.

I build the database using all the books registered under the category "Mathematics in the Early 1800", which includes all the titles related to mathematical developments before the year 1800. This list not only includes books published before 1800 but also books that were written before that year but the version available at the UC Berkeley Library was published after it. In all cases the year of the first publication of the book was looked for and replaced as the year of publication. It also includes compilations of several works from the same mathematician, whose separate works are not available at the Library. In those cases, these compilations were replaced by each book separately, using in each one the year it was first published. After cleaning the database, there is a total of 297 different mathematics books published in Europe between the years 1200 and 1700. There is no registry of books published before 1200.

3.3 Descriptive Statistics

The final database was built using the number of books published in each city only for cities in which population data was available for all centuries from 1200 to 1700. This delivers a total of 85 different cities, from which 29 have at least one book published there in the period 1200-1700. Other selection criteria were used, such as including all the cities with at least one population observation (which increased the number of books), but the results did not change substantially.

Table 1 presents the ranking of the thirty cities with the highest population each year within the final database, at the beginning of each century from 1200 to 1700. In general we can see a shift in the center of mass to northern Europe across time. In the year 1200 the list has several Spanish cities among the most populated, but most of them disappear over time. On the other hand, cities in the United Kingdom, Germany, and Northern Italy appear of scale through the ranking as time passes.

Other interesting fact from Table 1 is the appearance of Pisa in the year 1300. As *Liber Abaci* was published in 1202, Pisa is an interesting case of study. The book was published just after the previous population observation (1200) and ninety-eight

Table 1: Thirty Most Populated Cities (in thousands), 1200-1700

1200		1300		1400		1500		1600		1700	
	150	Granada	150	Paris	275	Paris	225	Paris	300	London	575
Paris	110	Paris	150	Brugge	125	Napoli	125	Napoli	275	Paris	500
Sevilla	80	Venezia	110	Venezia	100	Venezia	100	London	200	Napoli	300
Venezia	70	Genova	100	Genova	100	Praha	70	Venezia	151	Lisboa	180
Granada	60	Sevilla	90	Granada	100	Granada	70	Sevilla	135	Venezia	138
Cordoba	60	Napoli	60	Praha	95	Lisboa	65	Lisboa	130	Roma	135
Koeln	50	Cordoba	60	Sevilla	70	Tours	60	Palermo	105	Wien	114
Ieper	40	Koeln	54	Lisboa	55	Genova	58	Praha	100	Palermo	100
Smolensk	40	Palermo	51	Bologna	45	Palermo	55	Roma	100	Lyon	97
Roma	35	Salonika	50	London	45	Roma	55	Toledo	80	Marseille	90
Toledo	35	Siena	50	Napoli	45	Verona	50	Rouen	70	Sevilla	72
Bologna	35	Valencia	44	Toledo	45	London	50	Granada	69	Hamburg	70
Verona	33	Toledo	42	Salonika	42	Smolensk	50	Valencia	65	Granada	70
Genova	30	Bologna	40	Cordoba	40	Lyon	50	Tours	65	Antwerpen	67
Angers	30	Brugge	40	Malaga	40	Bordeaux	50	Smolensk	64	Genova	65
Salonika	30	Cremona	40	Ferrara	40	Bologna	50	Bologna	63	Bologna	63
Pavia	30	Malaga	40	Koeln	40	Orleans	50	Genova	63	Dublin	60
Speyer	30	Pisa	38	Valencia	36	Brescia	49	Salonika	50	Valencia	50
Napoli	30	Ferrara	36	Verona	35	Koeln	45	Wien	50	Rouen	50
Orleans	27	Padova	35	Rouen	35	Sevilla	45	Antwerpen	47	Praha	48
Metz	27	Montpellier	35	Pskov	35	Marseille	45	Augsburg	45	Liege	45
Ferrara	27	London	35	Cremona	35	Malaga	42	Marseille	45	Bordeaux	45
Valencia	26	Lisboa	35	Padova	34	Ferrara	42	Verona	45	Toulouse	43
Brugge	25	Rouen	35	Lyon	33	Valencia	42	Orleans	40	Wroclaw	40
Marseille	25	St-Omer	35	Roma	33	Rouen	40	Nuernberg	40	Salonika	40
London	25	Angers	33	Avignon	30	Cremona	40	Hamburg	40	Koeln	40
Cremona	25	Marseille	31	Liege	30	Nuernberg	38	Bordeaux	40	Nuernberg	40
Trier	25	Toulouse	30	Orleans	30	Cordoba	35	Magdeburg	40	Smolensk	40
Lyon	22	Arras	30	Bordeaux	30	Brugge	35	Toulouse	40	Padova	37
Erfurt	21	Pskov	30	Palermo	27	Toledo	32	Koeln	40	Brescia	35

years before the next one. Although this clearly does not implies that Pisa's economic development during the thirteenth century was due the implementation of Fibonacci's ideas, it rules out the possibility of endogeneity due reverse causality. After 1300 Pisa does not appear anymore among the thirty most populated cities in the sample. One possible explanation for this is that with time cities nearby Pisa also implemented the financial technologies developed by Fibonacci, taking Pisa back into its relative position among their "neighbors" (cities nearby), and seeing the rise of other Italian and French cities that might have been trade with Pisa.

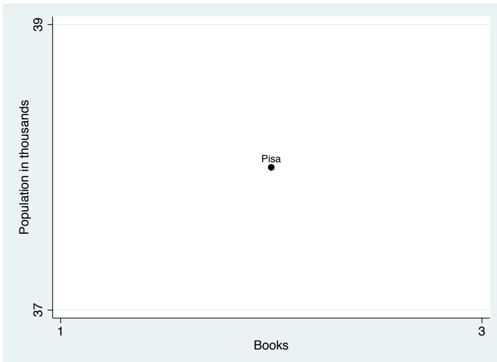
Figure 3 shows the relation between population and number of published books in the previous century for all the cities that had a positive number of published books each century. In general both the number of cities with published books and the average number of books increase among time. The dynamics also reflect the shift to the North of economic activity. For the latest centuries in the sample we have a higher presence of British, German, and French cities. On the other hand Italian cities, that dominate in the first panels, are not relevant in the last ones. Paris, London, and Venezia show a dominance over the rest of the cities in number of books.

4 Empirical Strategy

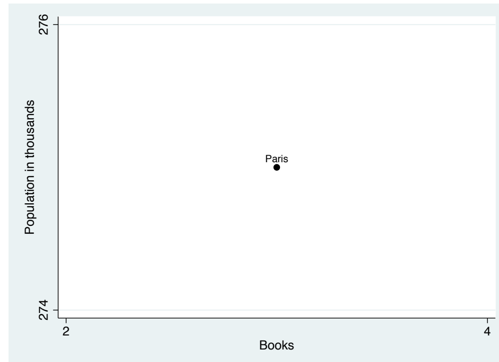
The main empirical strategy is the estimation of equation (1)

$$N_{i,t} = \alpha + \beta B_{i,[t-100,t-1]} + \gamma_i + \delta_t + \varepsilon_{i,t} \tag{1}$$

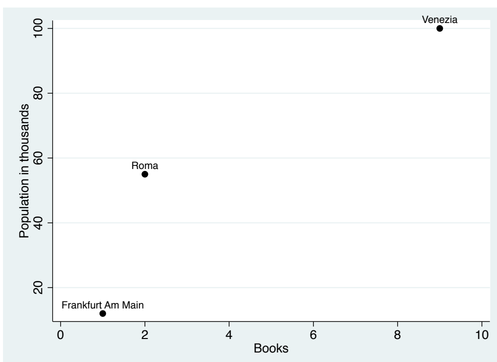
where $N_{i,t}$ refers to the population of city i at time t, measured in thousand habitants. $B_{i,[t-100,t-1]}$ is the number of mathematics books published in city i in the period between the years $t-100$ and $t-1$, both included. The terms γ_i and δ_t are city and time fixed effect, respectively, and $\varepsilon_{i,t}$ is the error term. The parameter of interest is β. It represents the potential effect of an additional mathematics book published during the period $[t-100, t-1]$ in a city on the population of that city in year t. We assume that this effect is through a higher knowledge of the financial mathematics techniques developed by Fibonacci. As an additional estimation, the term $B_{i,[t-100,t-1]}$



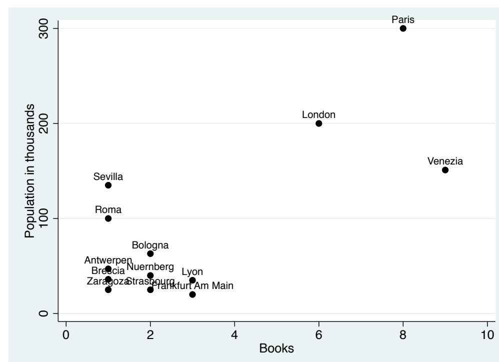
(a) 1300



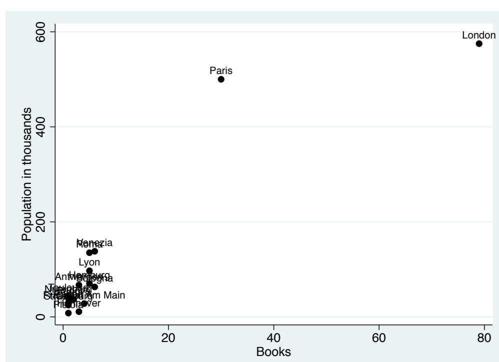
(b) 1400



(c) 1500



(d) 1600



(e) 1700

Figure 3: Relation between population and books published the previous centiry

is replaced by a dummy indicating if there were at least one book published:

$$N_{i,t} = \alpha + \mathbb{1}\left(B_{i,[t-100,t-1]} > 0\right) + \gamma_i + \delta_t + \varepsilon_{i,t} \tag{2}$$

where $\mathbb{1}(\cdot)$ is the indicator function. The benefits of this new equation are that it is parametrically more flexible and that the coefficient β can be interpreted as a treatment effect. The main limitation is that the scale effect of number of books is missing.

These equations are estimated for different times periods, all starting in 1200. The different periods are 1200 to 1300, 1200 to 1400, and so on until 1200 to 1700. The idea behind this strategy is to identify different dynamics in different periods of time. In particular, it is likely that the process of knowledge propagation changed with the use of the printing press. The change in availability of books given this technology led to an increase in book trade. Given this, it is likely that the city where the book was published is less important for the propagation of that knowledge after the printing press.

One first concern about this strategy is that books were not the only source of knowledge propagation in pre-industrial Europe. It is likely that most of the knowledge and practical techniques were transmitted orally, not written. This is even more likely in the case of commerce since by its own nature it implies that the businessman had to spend most of his time interacting with other commercial partners, which obviously makes the communication more likely. Moreover, it is also possible that books that were published in one city were read in other, which makes the number of published books a bad measure of exposure to new financial technologies. This last problem becomes more likely after the widespread of the printing press in the sixteenth century. As books were massively produced, book trade increased and the likelihood of one idea being transmitted through books from one city to another increased. However, as long as the variation in mathematics books is itself a relevant source of variation within the total exposure to financial technologies in different cities, then the estimation of β should identify at least a portion of the effect of that exposure. This is similar to a measurement error problem, so the estimation of the coefficient β should be be downward biased.

A second potential problem in the estimation is endogeneity due reverse causality.

It is possible richer cities attract mathematicians, either because universities were established in richer cities or because cities with more resources offered wages to scientists to develop their time to do research. In this case, it is possible that the estimation yields an upward biased estimation of β. Although the number of books is before the observation, the fact that the gap between population observations is one hundred years makes it possible for a city to grow within a century, generating mathematics books within the same century but after the growth.

Again, this problem is more likely to bias the result after the sixteenth century and the printing press. With the invention of the printing press the business of book production was born. As access to the printing press as a production technology required a high initial investment (see Mokyr (2005)), richer cities were more likely to enter the books business. Then, cities with higher income would have a higher number of published books.

To solve the reverse causality problem, variations of the two specifications described above are estimated. Instead of using the number of books between $t-100$ and $t-1$ and it respective dummy, β is estimated using the number of books published only in the first half of each century. Thus, $B_{i,[t-100,t-1]}$ is replaced by $B_{i,[t-100,t-51]}$, and $\mathbb{1}\left(B_{i,[t-100,t-1]} > 0\right)$ by $\mathbb{1}\left(B_{i,[t-100,t-51]} > 0\right)$. Although this does not solve the problem completely, it makes it less likely for the reverse causality to be the prominent effect in the estimation of β since it would require for cities to grow systematically higher in the first half of each century than in the second. There is no historical background to support that.

As a second form to address the reverse causality problem I run falsification exercises for all the previous regressions. For each of the previous equations, I replace the value of the variable referring to books for the lead that same variable (the variable in the next period). A statistically significant estimation of β in these regressions would mean that there is reverse causality, since the level of the dependent variable is correlated with future levels of the dependent variable.

5 Main Results

Results of the estimation of equations (1) and (2) are presented in Table 2. All estimations but the ones with the shortest possible time period present positive and statistically significant estimations of β.

Table 2: Estimations for all Books

	(1)	(2)	(3)	(4)	(5)
VARIABLES	1200 - 1300	1200 - 1400	1200 - 1500	1200 - 1600	1200 - 1700
$B_{i,[t-100,t-1]}$	7.071	35.10***	4.506**	9.561***	6.902***
	(10.10)	(5.150)	(1.891)	(1.430)	(0.290)
R-squared	0.116	0.260	0.071	0.207	0.610
$\mathbb{1}\left(B_{i,[t-100,t-1]} > 0\right)$	14.14	78.45***	30.98***	33.44***	44.64***
	(20.20)	(13.54)	(8.286)	(6.014)	(7.244)
R-squared	0.116	0.212	0.100	0.178	0.160
Observations	170	255	340	425	510
Number of Cities	85	85	85	85	85

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

The non-significant result in the estimation for the period 1200-1300 is expected since there is not enough variation in number of books. Moreover, the statistically significant results for the period 1200-1400 is surprising given that the variation in number of books is still very small (only two cities with positive number of books). Results show that there exists a relation between economic prosperity and number of mathematics books in the previous century. This relation is robust to different time periods (except for the 1200-1300, as explained before). Regarding the precision of the results, we can see that when regressing on number of books the estimations become more precise when time advances (since there is more variation in number of books).

This is gain in precision is less clear in the case in the case of the regressions that include the dummy variable instead of the number of books. Moreover, the change on the standard deviations over time periods is higher for number of books. In the case when the dummy variable is used the standard deviations are not monotone on time periods.

In order to avoid the endogeneity problems regressions are estimated counting only the books published during the first half of each tear, as described above. Results of these estimations are presented in Table 3. Results are consistent with the previous ones: all estimations of β are positive and statistically significant besides the ones. The reason why columns (1) and (2) present the same results in tables 2 and 3 is that all books in the period 1200-1400 were published in the first half of their respective century.

Table 3: Estimations for Books written during the first half of each century

	(1)	(2)	(3)	(4)	(5)
VARIABLES	1200 - 1300	1200 - 1400	1200 - 1500	1200 - 1600	1200 - 1700
$B_{i,[t-100,t-51]}$	7.071	35.10***	25.17***	14.72***	31.31***
	(10.10)	(5.150)	(4.626)	(3.112)	(1.479)
R^2	0.116	0.260	0.150	0.158	0.557
$\mathbb{1}\left(B_{i,[t-100,t-51]} > 0\right)$	14.14	78.45***	33.67***	26.64***	60.43***
	(20.20)	(13.54)	(9.244)	(7.416)	(8.693)
R^2	0.116	0.212	0.097	0.135	0.179
Observations	170	255	340	425	510
Number of Cities	85	85	85	85	85

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

An interesting result is that almost all the estimations are quantitatively higher when only books published in the first half of the century are included. The only case in which the result is lower is the specification with the dummy variable in the 1200-1600 period. This finding supports the idea that mathematic books were a way to spread financial knowledge and that this knowledge increased income. The reason for this is that we measure a higher effect from books that were published earlier in the century, since they spend more time published before the new measure of population is taken. This gives the book a longer period of time to "act", which leads to a higher economic growth during the century. A simple analogous of this is that a change in the interest rate yields a higher higher return at the end of the period the sooner it happens.

The bigger changes between using all the books and using only the books published in the first half of the century are in the specifications that use the number of books. Here, the estimations of Table 3 are around four times the results of Table 2 (taking into account only the periods in which the results differ). If we assume that books' publications are uniformly distributed across time, this result implies that the effect of financial techniques over wealth is not linear in time. A closer behavior could be for it to generate a compound return over time.

Finally, as a falsification exercise all the previous regressions are estimated replacing the variable that measure number of books (or dummy for positive number of books) by the lead of the same variable (the variable in the next period). Therefore, in the previous regressions $B_{i,[t-100,t-1]}$ is replaced with $B_{i,[t,t+99]}$, and $B_{i,[t-100,t-51]}$ with $B_{i,[t,t-49]}$. Results are presented in tables 4 and 5 respectively.

Results of the falsification exercise reflect neatly the change in the dynamics of books publication and information flow before and after the printing press. The falsification exercises for the periods before the sixteenth century are not statistically significant[9]. This suggest that the potential reverse causality is not present, and the effect found in the previous results is in fact a causal effect of financial technology over economic growth.

Finally, falsification results for the time periods that include the presence of the printing press show that the estimations are biased, and that book publication is not an adequate measure of exposition to financial technologies. The statistically significant results on the falsification exercises suggest that the presence of reverse causality is

[9] The only significant result is the coefficient of $B_{i,[t,t-49]}$ in column (2) of Table 5. However, it is statistically significant only at the 10% level and its sign is the opposite than the one expected. This suggest that its significance is only a case of type I error.

Table 4: Falsification Exercise for all Books

	(1)	(2)	(3)	(4)	(5)
VARIABLES	1200 - 1300	1200 - 1400	1200 - 1500	1200 - 1600	1200 - 1700
$B_{i,[t,t+99]}$	5.465	-1.012	1.957	2.145***	3.624***
	(5.556)	(2.113)	(1.248)	(0.262)	(0.405)
R^2	0.121	0.056	0.059	0.251	0.231
$\mathbb{1}\left(B_{i,[t,t+99]} > 0\right)$	9.500	-9.999	4.723	10.61**	24.98***
	(14.20)	(9.371)	(5.081)	(4.955)	(6.709)
R^2	0.116	0.061	0.053	0.114	0.113
Observations	170	255	340	425	510
Number of Cities	85	85	85	85	85

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 5: Falsification Exercise for Books written during the first half of each century

	(1)	(2)	(3)	(4)	(5)
VARIABLES	1200 - 1300	1200 - 1400	1200 - 1500	1200 - 1600	1200 - 1700
$B_{i,[t,t+49]}$	5.465	-9.675*	0.199	9.854***	20.98***
	(5.556)	(5.353)	(2.617)	(1.276)	(1.263)
R^2	0.121	0.072	0.050	0.238	0.448
$\mathbb{1}\left(B_{i,[t,t+49]} > 0\right)$	9.500	-11.66	0.0555	15.25**	41.26***
	(14.20)	(10.44)	(6.141)	(5.996)	(7.754)
R^2	0.116	0.061	0.050	0.119	0.142
Observations	170	255	340	425	510
Number of Cities	85	85	85	85	85

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

stronger after the invention of the printing press, and we cannot recover the parameter of interest. As explained before, this is more likely due the high costs of acquiring the printing press technologies, which make it a resource available only for rich cities, which can make a profitable investment out of this. This generates an endogenous relation between income and number of books.

The bias in the post-the printing press era results do not necessarily mean that acquisition of financial technologies did not had an effect over income. It is reasonable to think that the increase in books trade increased the diffusion of information across cities. Then, the city where a book was published is not an indicator of that specific city receiving the information in that book and the rest receiving nothing. Since the book is traded in other cities, several cities receive the information. Therefore the falsification exercise, more than suggesting that the effect of financial technology is not real, suggests that after the invention of the printing press the number of published books in a city is not adequate as a measure of information acquisition.

6 Conclusions

Technological change is one of the oldest and more relevant topics in economics. The question about how to increase the income of a person or society without the necessity of more factor accumulation will probably always be relevant, both in political and academic discussions. In this work I try to address a specific type of technological change: changes in financial technologies. Although financial technologies do not make an economy more productive in the literal sense (the economy is not capable of producing more goods with the same resources), it does increase its income without the necessity of more resources by increasing the rate of return of capital.

Without a doubt the greatest change in financial techniques in pre-industrial Europe was the publication of Fibonacci's book *Liber Abaci*. Here, the mathematician introduced the Hindu-Arabic numeric system to Europe and its arithmetic, which is easier than the Roman system for business applications. Moreover, he introduced techniques to operate with fractions, and developed calculations of financial elements like exchange rates, division of profits, interest rates, and present values. Although Fibonacci is well known for his work in mathematics, several consider that his contribution to finance is even greater than his contribution for which he is famous.

By using the number of mathematics books published in each city as a measure of exposure to Fibonacci's ideas in that place, I estimate the effect of the adoption of this new financial technologies on economic development. As there is no city-level economic income data for the pre-industrial period, I use instead the population data from Bairoch's database. The approach used in this paper presents two main caveats. First, it is possible that the publications of books account for a little part of the variation in exposure to these new financial techniques, specially if many information was transmitted personally or if books were published in one city but read in others. Second, there is possible reverse causality. Both problems become more relevant after the widespread of the printing press in the sixteenth century.

Results suggest that there mathematics books were a source of financial knowledge before the printing press. Addressing the aforementioned problems, estimations suggest that before the printing press the acquisition of financial technology through mathematics books generated a positive effect on economic welfare, which accumulated over time as a compound rate. On the other hand results suggest that the use of books publication is not an adequate measure of exposure to financial technologies under the presence of the printing press.

Several authors suggest that new ideas, activities, and social groups in pre-industrial Europe were the fundamentals for the Industrial Revolution. The findings presented in this paper suggest that the advances in financial techniques were part of these ideas.

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